Quantum Mechanics

Lecture #2

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Postulates of Quantum Mechanics:

"Postulates of Quantum Mechanics"

Postulate 1 :- It is stated that "The state of quantum mechanical system is completely specified by a function $\psi(r,t)$ that depends on coordinates of particle(s) and on time. This function, called "wave fn." or "state function", has important property that $\psi^*_{(r,t)}\psi_{(r,t)}\,d\tau$ is the probability that the particle lies in the volume element $d\tau$ located at r at time t."

→ This wave function should be single valued, finite and continuous.

b) It states that nature of wave function ψ is such that

$$\psi\psi^*\,dx\,dy\,dz = \psi\psi^*\,d\tau$$

should represent the probability of finding particle in volume $d\tau$. ψ^* is complex conjugate of ψ and

$$\psi\psi^* = \psi^2 \qquad \because \text{Probability is always +ive.}$$

• If we consider a volume which encloses a particle then

$$\int \psi\psi^*\,d\tau = 1$$

• For many particles

$$\int \psi\psi^*\,d\tau_1, d\tau_2, d\tau_3 = 1$$

Cont.

where integration is taken over entire space.

⇒ This postulate tells us about 'probability' and 'wave function'.

Postulate 2 :- This postulate is stated as
"To every observable in classical mechanics there corresponds a linear Hermition operator in quantum mechanics."

- Observables like energy of a system, position of a particle and momentum etc.
- If we denote an operator by 'Q' and 'q' as physical observable associated then measurement of 'q' gives a result which is one of the eigen value of eigen equation.

$$Q_n \psi_n = q_n \psi_n$$

$\because Q^* = Q$

$$\int \psi_n^* Q \psi_n \, d\tau = \int (Q \psi_n)^* \psi_n \, d\tau$$

Postulate 3 :- "According to this postulate, average value $\bar{Q}$ of a quantity represented by an operator Q is represented by

$$\bar{Q} = \int \psi^* Q \psi \, d\tau$$

As

$$\langle x \rangle = \int \psi^* \hat{x} \psi \, d\tau$$

Orthonormal set of Eigen/Wave function:

Orthonormal set of Eigen / wave function :-

"An orthogonal function whose inner product with itself is unity is called orthonormal function."

- It is the combination of normality and orthogonality.

- Suppose we have a set of Eigen functions i-e

$$\psi_1 , \psi_2 , \psi_3 , --- , \psi_n$$

This is the representation of 'n' eigen functions. They would behave like normalized set of Eigen function. i-e if

$$\int \psi_n \psi_n^* \, d\tau = 1$$

This is called "normalization condition."
If both states are change i-e if ψ_m and ψ_n are present than

$$\int \psi_m \psi_n \, d\tau = 0 \qquad m \neq n$$

or

$$\int \psi_m \psi_n \, d\tau = 1 \qquad m = n$$

- It could be zero or 1.

Cont.

- The whole situation described is called 'orthonormality of eigen function.'

i.e

δ_{mn} or $\delta_m \delta_n = 0$ $\quad m \neq n$

δ_{mn} or $\delta_m \delta_n = 1$ $\quad m = n$

where

'δ_{mn}' is called '<u>kronekar</u> <u>delta</u>'

- If wave function is normalized and orthogonalized then the fn. is <u>orthonormal</u>.

Degeneracy:

Degeneracy:-

"If we have ψ_i and ψ_k and both of them correspond to same energy then it is called degeneracy."

Conversly,

"Two or more different states of a quantum mechanical system are said to be degenerate if they give the same value of energy upon measurement"

The no. of different state corresponding to a particular energy level is known as "degree of degeneracy of level."

Example:- If ψ_1 and ψ_2 are two wave fns. then both of them contain same energy

Cont.

- These wave fns. are called "degenerate wave fns."

In 'p' level 6 electrons are present. So,

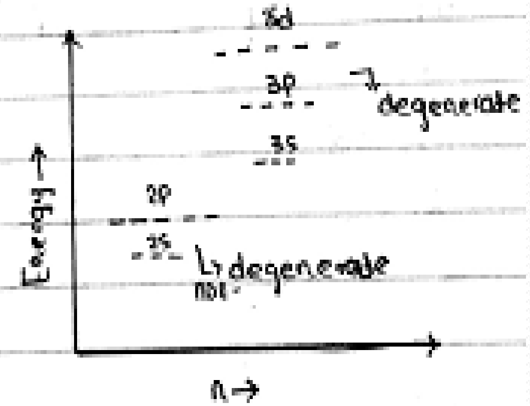
$$^6p = p_x, p_y, p_z, p_{x'}, p_{y'}, p_{z'}$$

- These are all "degenerate energy levels."
- Also 3s, 3p and 3d are degenerate energy levels.

⇒ "If they have different energy levels then they are called non-degenerate energy levels."

e.g.

1s, 2s, 2p



Complete set of wave/Eigen functions:

Complete set of wave/Eigen functions :-

Suppose that a linear combination of members of orthogonal set of eigen function of an operator is

$$\sum_{i=1}^{\infty} C_i \psi_i \quad \text{—— (i)}$$

where C_i are the complex numbers. or called complex coefficient

If the given series converges

$$f = \sum_{i=1}^{\infty} C_i \psi_i \quad \text{—— (ii)}$$

Cont.

- A large no. of eigen functions can be represented as linear combination of eigen functions.

In case of set of eigen functions

$$\psi = \sum_{j=1}^{\infty} B_j \psi_j$$

or

$$\psi = \sum_{r=1}^{n} K_r \psi_r \quad \text{———} \quad (iii)$$

where 'B_j' and 'K_r' are coefficients if wave fns. are different for (different) each particles.

Eq. (iii) is the representation of complex set of wave / Eigen function.

* Purpose :-

If we have different type of functions then we use this notation.

* Properties:-

- It should be convergeable.
- It should correspond with operation.
- It should be orthonormal.